What is claimed as new and desired to be secured by Letters Patent of the United States is:

CLAIMS

1. A computer graphics system for generating a pixel value for a pixel in an image, the pixel being representative of a point in a scene as recorded on an image plane of a simulated camera, the computer graphics system comprising:

A. a sample point generator configured to generate a set of sample points representing at least one simulated element of the simulated camera, the sample points representing elements of a Hammersley sequence; and

B. a function evaluator configured to generate at least one value representing an evaluation of said selected function at one of the sample points generated by said sample point generator, the value generated by the function evaluator corresponding to the pixel value.

2. A computer graphics system as defined in claim 1 in which the sample point generator is configured to generate sample position points x_i representing jittered sample point positions on a subpixel grid for at least one pixel on the image plane for use by the function evaluator in evaluating the selected function.

3. A computer graphics system as defined in claim 2 in which the sample point generator is configured to generate the sample position points x_i in accordance with

$$\begin{aligned} x_i &= \left(s_x + \Phi_2(k), s_y + \Phi_2(j)\right) \\ &= \left(s_x + \frac{\sigma(k)}{2^n}, s_y + \frac{\sigma(j)}{2^n}\right) \end{aligned}$$

where (s_x, s_y) are subpixel coordinates mapped onto strata coordinates $(j,k) := \left(s_x \bmod 2^n, s_y \bmod 2^n\right)$, and instance number "i" corresponds to

$$i = j\,2^n + \sigma(k)$$

where integer permutation $\sigma(k) := 2^n\ \Phi_2(k)$ for $0 \leq k < 2^n$ for "n" a selected integer, and $\Phi_b(q)$ is the radical inverse function of "q" in base "b."

4. A computer graphics system as defined in claim 2 in which the simulated camera is to be provided with a shutter, and the sample point generator is configured to generate, for at least one sample point position on a subpixel for at least one pixel on the image plane, sample time points $t_{i,j}$ representing "j" points in time during a time interval t_0 to t_0+T during which the shutter is to be open for use by the function evaluator in evaluating the selected function. .

5. A computer graphics system as defined in claim 4 in which the sample point generator is configured to generate the sample time points $t_{i,j}$ in accordance with

$$t_{i,j} \quad := \quad t_0 + \left(\Phi_3(i) \oplus \frac{j}{N_T} \right) \cdot T$$

where N_T corresponds to a predetermined number of times at which sample time points $t_{i,j}$ are to be generated and $\Phi_b(q)$ is the radical inverse function of "q" in base "b."

6. A computer graphics system as defined in claim 4 in which the simulated camera is to be provided with a lens, and the sample point generator is configured to generate, for at least one sample point position and at least one sample time point, sample lens position points $y_{i,j,k}$ representing "k" points on the lens for use by the function evaluator in evaluating the selected function.

7. A computer graphics system as defined in claim 6 in which the sample point generator is configured to generate the sample lens points in accordance with

$$y_{i,j,k} \quad = \quad \left(\left(\Phi_5(i+j,\sigma_5) \oplus \frac{k}{N_L} \right), \left(\Phi_7(i+j,\sigma_7) \oplus \Phi_2(k) \right) \right)$$

where $\Phi_b(q,\sigma)$ is the scrambled radical inverse function of "q" in base "b" and "$\oplus$" refers to addition modulo a predetermined value.

8. A computer graphics system as defined in claim 7 in which the predetermined value is "one."

9. A computer graphics method for generating a pixel value for a pixel in an image, the pixel being representative of a point in a scene as recorded on an image plane of a simulated camera, the computer graphics method comprising:

A. a sample point generating step in which a set of sample points are generated representing at least one simulated element of the simulated camera, the sample points representing elements of a Hammersley sequence; and

B. a function evaluation step in which at least one value is generated representing an evaluation of said selected function at one of the sample points generated by said sample point generator, the value generated by the function evaluator corresponding to the pixel value.

10. A computer graphics method as defined in claim 9 in which the sample point generating step includes the step of generating sample position points x_i representing jittered sample point positions on a subpixel grid for at least one pixel on the image plane for use by the function evaluator in evaluating the selected function.

11. A computer graphics method as defined in claim 10 in which the sample point generating step includes the step of generating the sample position points x_i in accordance with

$$\begin{aligned} x_i &= \left(s_x + \Phi_2(k), s_y + \Phi_2(j)\right) \\ &= \left(s_x + \frac{\sigma(k)}{2^n}, s_y + \frac{\sigma(j)}{2^n}\right) \end{aligned}$$

where (s_x,s_y) are subpixel coordinates mapped onto strata coordinates $(j,k) := (s_x \bmod 2^n, s_y \bmod 2^n)$, and instance number "i" corresponds to

$$i = j\,2^n + \sigma(k)$$

where integer permutation $\sigma(k) := 2^n\ \Phi_2(k)$ for $0 \le k < 2^n$ for "n" a selected integer, and $\Phi_b(q)$ is the radical inverse function of "q" in base "b."

12. A computer graphics method as defined in claim 10 in which the simulated camera is to be provided with a shutter, and the sample point generating step includes the step of generating, for at least one sample point position on a subpixel for at least one pixel on the image plane, sample time points $t_{i,j}$ representing "j" points in time during a time interval t_0 to t_0+T during which the shutter is to be open for use by the function evaluator in evaluating the selected function. .

13. A computer graphics method as defined in claim 12 in which the sample point generating step includes the step of generating the sample time points $t_{i,j}$ in accordance with

$$t_{i,j} := t_0 + \left(\Phi_3(i) \oplus \frac{j}{N_T}\right) \cdot T$$

where N_T corresponds to a predetermined number of times at which sample time points $t_{i,j}$ are to be generated and $\Phi_b(q)$ is the radical inverse function of "q" in base "b."

14. A computer graphics method as defined in claim 13 in which the simulated camera is to be provided with a lens, and the sample point generating step includes the step of generating, for at least one sample point position and at least one sample time point, sample lens position points $y_{i,j,k}$ representing "k" points on the lens for use by the function evaluator in evaluating the selected function.

15. A computer graphics method as defined in claim 14 in which the sample point generating step includes the step of generating the sample lens points in accordance with

$$y_{i,j,k} = \left(\left(\Phi_5(i+j,\sigma_5) \oplus \frac{k}{N_L} \right), \left(\Phi_7(i+j,\sigma_7) \oplus \Phi_2(k) \right) \right)$$

where $\Phi_b(q,\sigma)$ is the scrambled radical inverse function of "q" in base "b" and "$\oplus$" refers to addition modulo a predetermined value.

16. A computer graphics method as defined in claim 15 in which the predetermined value is "one."

17. A computer program product for use in connection with a computer to provide a computer graphics system for generating a pixel value for a pixel in an image, the pixel being representative of a point in a scene as recorded on an image plane of a simulated camera, the computer program product comprising a machine readable medium having encoded thereon

A. a sample point generator module configured to enable the computer to generate a set of sample points representing at least one simulated element of the simulated camera, the sample points representing elements of a Hammersley sequence; and

B. a function evaluator module configured to enable the computer to generate at least one value representing an evaluation of said selected function at one of the sample points generated by said sample point generator module, the value generated by the function evaluator corresponding to the pixel value.

18. A computer program product as defined in claim 17 in which the sample point generator module is configured to enable the computer to generate sample position points x_i representing jittered sample point positions on a subpixel grid for at least one pixel on the image plane for use by the function evaluator in evaluating the selected function.

19. A computer program product as defined in claim 18 in which the sample point generator module is configured to enable the computer to generate the sample position points x_i in accordance with

$$
\begin{aligned}
x_i &= \left(s_x + \Phi_2(k), s_y + \Phi_2(j)\right) \\
&= \left(s_x + \frac{\sigma(k)}{2^n}, s_y + \frac{\sigma(j)}{2^n}\right)
\end{aligned}
$$

where (s_x, s_y) are subpixel coordinates mapped onto strata coordinates $(j,k) := \left(s_x \bmod 2^n, s_y \bmod 2^n\right)$, and instance number "i" corresponds to

$$
i = j\,2^n + \sigma(k)
$$

where integer permutation $\sigma(k) := 2^n\, \Phi_2(k)$ for $0 \le k < 2^n$ for "n" a selected integer, and $\Phi_b(q)$ is the radical inverse function of "q" in base "b."

20. A computer program product as defined in claim 18 in which the simulated camera is to be provided with a shutter, and the sample point generator module is configured to enable the computer to generate, for at least one sample point position on a subpixel for at least one pixel on the image plane, sample time points $t_{i,j}$ representing "j" points in time during a time interval t_0 to t_0+T during which the shutter is to be open for use by the function evaluator in evaluating the selected function.

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21. A computer program product as defined in claim 20 in which the sample point generator module is configured to enable the computer to generate the sample time points $t_{i,j}$ in accordance with

$$
t_{i,j} := t_0 + \left(\Phi_3(i) \oplus \frac{j}{N_T}\right) \cdot T
$$

where N_T corresponds to a predetermined number of times at which sample time points $t_{i,j}$ are to be generated and $\Phi_b(q)$ is the radical inverse function of "q" in base "b."

22. A computer program product as defined in claim 20 in which the simulated camera is to be provided with a lens, and the sample point generator module is configured to enable the computer to generate, for at least one sample point position and at least one sample time point, sample lens position points $y_{i,j,k}$ representing "k" points on the lens for use by the function evaluator in evaluating the selected function.

23. A computer program product as defined in claim 22 in which the sample point generator module is configured to enable the computer to generate the sample lens points in accordance with

$$y_{i,j,k} = \left(\left(\Phi_5(i+j,\sigma_5) \oplus \frac{k}{N_L}\right), \left(\Phi_7(i+j,\sigma_7) \oplus \Phi_2(k)\right)\right)$$

where $\Phi_b(q,\sigma)$ is the scrambled radical inverse function of "q" in base "b" and "$\oplus$" refers to addition modulo a predetermined value.

24. A computer program product as defined in claim 23 in which the predetermined value is "one."